Attempt All the Following Questions

1- Derive an expression for the boundary work for a polytropic process. (5 Marks)

2- A gas is compressed from an initial volume of 0.42 m$^3$ to a final volume of 0.12 m$^3$. During the quasi-equilibrium process, the pressure changes with volume according to the relation $P = aV + b$, where $a = -1000$ kPa/m$^3$ and $b = 500$ kPa. Calculate the work done during this process (a) by plotting the process on a P-V diagram and finding the area under the process curve and (b) by performing the necessary integrations. (5 Marks)

3- Water at a pressure of 4 MPa and temperature of 430$^\circ$C. Find its specific volume using
   a- Water tables
   b- Ideal gas equation
   c- Real gas equation
   d- Calculate the errors in both ideal and real gas equations. (10 Marks)

With Best Wishes

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Model (A)

\[ W_b = \frac{P_2 V_2 - P_1 V_1}{1-n} \quad \rightarrow \text{See Sec. notes} \]

\[ V_1 = 0.42 \text{ m}^3 \]
\[ V_2 = 0.12 \text{ m}^3 \]
\[ P = aV + b \]
\[ a = -1000 \text{ kPa/m}^3 \]
\[ b = 500 \text{ kPa} \]

\[ P_1 = a V_1 + b \]
\[ P_1 = (-1000)(0.42) + 500 \]
\[ P_1 = 80 \text{ kPa} \]

\[ P_2 = (-1000)(0.12) + 500 \]
\[ P_2 = 380 \text{ kPa} \]

\[ W_b \]

a) Area under process
b) Integration

\[ W_b = \text{Area of Rectangle} + \text{Area of Triangle} \]
\[ = (80)(0.42-0.12) + \frac{1}{2}(0.42-0.12)(380-80) \]
\[ = 69 \text{ kJ} \]

Because it (input) \[ W_b = -69 \text{ kJ} \]
b) \[ W_b = \int_1^2 P \, dv \]
\[ = \int_1^2 a \, v + b \, dv \]
\[ = \frac{a}{2} v^2 + b v \bigg|_1^2 \]
\[ W_b = \frac{a}{2} (v_2^2 - v_1^2) + b(v_2 - v_1) \]
\[ = \frac{-1000}{2} \left( 0.12^2 - 0.42^2 \right) + 500(0.12 - 0.42) \]
\[ W_b = -69 \text{ kJ} \]

3. \[ P = 4 \text{ MPa} \]
\[ T = 430 \text{°C} \]

a) water Table

b) ideal gas eq.

c) real gas eq.

d) error

Using Table (A-5)

@ \[ P = 4 \text{ MPa} \]

\[ T_{\text{sat}} = 250.35 \text{°C} \]

\[ T_{\text{gim}}(430) > T_{\text{sat}}(250.35) \]

\[ \therefore \text{The state is superheated.} \]
Using Table (A-6) @ $P = 4\text{MPa}$

\begin{align*}
T & \quad \nu \\
400 & \quad 0.07343 \\
450 & \quad 0.08004
\end{align*}

By interpolation:

\[
\frac{430 - 400}{450 - 400} = \frac{\nu - 0.07343}{0.08004 - 0.07343}
\]

\[
\nu = 0.0773 \text{ m}^3/\text{kg}
\]

b) Using ideal gas eq.

\[
R = 0.4615 \text{ kJ/kmol-K}
\]

\[
P \nu = RT \Rightarrow \nu = \frac{(0.4615)(430+273)}{4 \times 10^3}
\]

\[
\nu = 0.0811 \text{ m}^3/\text{kg}
\]

c) Using real gas eq.

\[
\begin{align*}
P_r & = \frac{P}{P_c} = 0.18 \\
T_r & = \frac{T}{T_c} = \frac{430+273}{647.1} = 1.08
\end{align*}
\]

Using chart:

\[
Z = 0.95 \quad \nu = 0.0770 \text{ m}^3/\text{kg}
\]
Error = \left[ \frac{V_{\text{calculated}}}{V_{\text{table}}} - 1 \right] \times 100\%

= \left( \frac{0.0811}{0.0773} - 1 \right) \times 100\%

Error = 4.91\%.

Error = \left[ \frac{V_{\text{calculated}}}{V_{\text{table}}} - 1 \right] \times 100\%

= \left( \frac{0.0777}{0.0773} - 1 \right) \times 100\%

Error = -0.388\%.